[10:30] Performance Analysis for PAM (Lecture Slide 15-11)

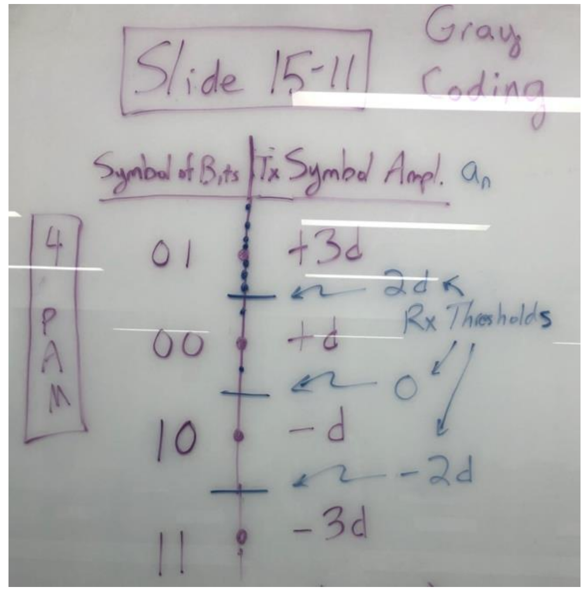
- $P(\text{error})$ depends on noise power σ^2, symbol period T_{sym}, symbol amplitude d
- In practice, error correcting codes are used to tolerate a certain probability of error
- Gray Coding: choose encoding so that adjacent symbols have one bit of difference

4 PAM example:

Symbol of bits	TX Symbol Amplitude
01	$+3d$
00	$+d$
10	$-d$
11	$-3d$

$$x_n = a_n + \underbrace{v_n}_{\substack{\text{RV} \\ N\left(0,\frac{\sigma^2}{T_{sym}}\right)}} \leftarrow \text{assuming only impairment is additive thermal noise}$$

$$\underbrace{P_{\text{PAM}}(\text{error})}_{\text{lower bound}} = \underbrace{\frac{2(M-1)}{M}}_{\text{range } [1,2)} \underbrace{Q\left(\overbrace{\frac{d}{\sigma}\sqrt{T_{sym}}}^{\text{depends on SNR}}\right)}_{\text{exponential decay+}}$$



Example received symbol amplitudes shown for transmitted symbol amplitude $3d$

[10:55] Performance analysis for QAM (Lecture Slide 15-12)

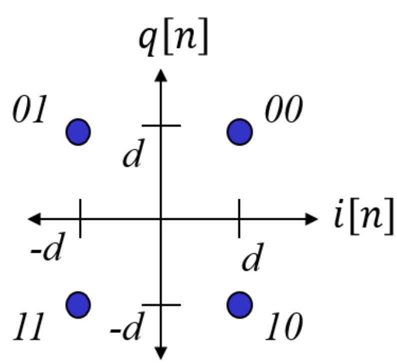
- Treat symbol amplitude as a complex number
    - Real part: in phase
    - Imaginary: quadrature
- Transmitted signal is $s(nT_{sym}) = a_n + jb_n = (2i-1)d + j(2k-1)d$

  $(i, k \in \{-1,0,1,2\}$ for 16-QAM)

4 QAM constellation

- Quadrature $q[n]$ (PAM symbol amplitude)
- In-Phase $i[n]$ (PAM symbol amplitude)

Symbol of bits	$i[n]$	$q[n]$
00	d	d
01	$-d$	d
10	d	$-d$
11	$-d$	$-d$



[11:05] Performance analysis for 16-QAM (Lecture Slides 15-13 to 15-15)

- Optimal decision algorithm: minimize Euclidian distance
- For rectangular constellation, thresholding is equivalent to minimizing distance
- Threshold at multiples of $2d$ on each axis (midpoint between adjacent values)
- Assumption: noise independent in in-phase and quadrature
- Assumption: all symbols in the constellation are equally likely

Three different types of region are possible

- Type 1: Finite in both dimensions

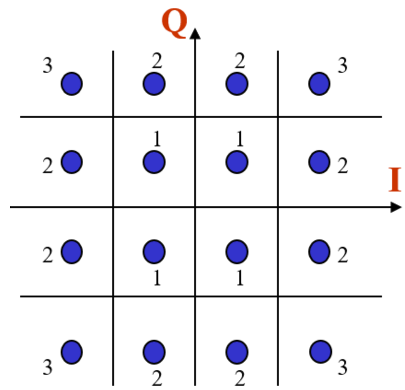
$$P(\text{correct}|T_1) = \left(1 - 2Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)^2$$

- Type 2: Finite in one dimension

$$P(\text{c}|T_2) = \left(1 - Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)\left(1 - 2Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)$$

- Type 3: Quarter plane (infinite in both)

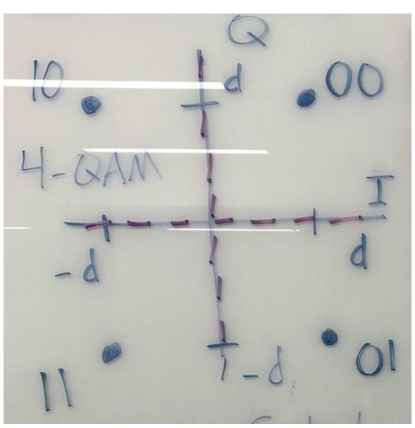
$$P(\text{c}|T_3) = \left(1 - Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)\right)^2$$

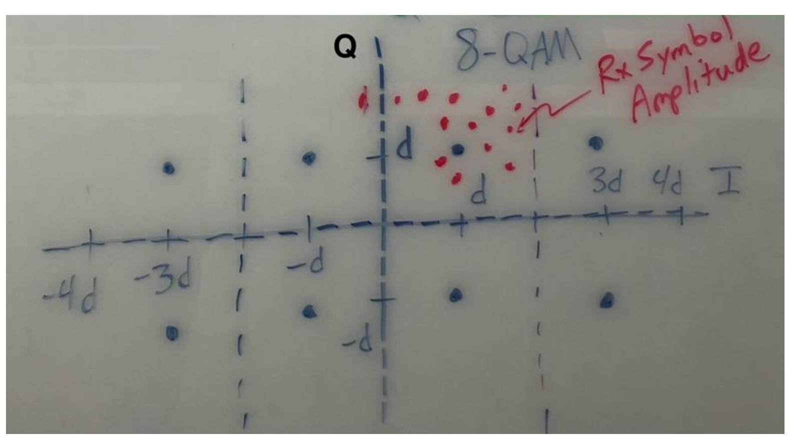


$$P_{16\text{ QAM}}(c) = \frac{4}{16}P(\text{c}|T_1) + \frac{4}{16}P(\text{c }|T_2) + \frac{8}{16}P(\text{c}|T_3)$$

[11:25] QAM constellations (Lecture Slide 15-15)

	Rectangular decision region type		
QAM	**Type 1**	**Type 2**	**Type 3**
4	0	0	4
8	0	4	4
16	4	8	4
32	12	16	4
64	36	24	4





[11:35] Power analysis (Lecture Slide 15-16)

- Assume each symbol is equally likely and the energy in pulse shape is one.
- Lower peak to average power ratio is preferred for power amplifier design
- 4-PAM constellation
  - Amplitudes: $\{-3d, -d, d, 3d\}$
  - Total power $= 9d^2 + d^2 + d^2 + 9d^2 = 20d^2$
  - Average power $= \frac{1}{4}$ Total power $= 5d^2$
  - Peak to average power ratio $= 1.8$
- 4-QAM constellation
  - Amplitudes: $\{-d - jd, -d + jd, d + jd, d - jd\}$
  - Total power $= 2d^2 + 2d^2 + 2d^2 + 2d^2 = 8d^2$
  - Average power $= \frac{1}{4}$ Total power $= 2d^2$
  - Peak to average power ratio $= 1.0$
- Higher peak-to-average-power ratio makes the design and part cost for the power amplifier in the analog/RF processing chain more expensive
- Moving from 4-PAM to 4-QAM improves average power, peak power, and ratio

$$SNR = \frac{\text{signal power}}{\text{noise power}}$$

4-PAM	**4-QAM**
$SNR = \frac{5d^2}{\sigma^2}$ (before matched filter) $SNR = \frac{5d^2}{\sigma^2 / T_{sym}}$ (after matched filter) $\sqrt{SNR} = \sqrt{5}\frac{d}{\sigma}\sqrt{T_{sym}}$ $\frac{d}{\sigma}\sqrt{T_{sym}} = \sqrt{\frac{SNR}{5}}$	$\frac{d}{\sigma}\sqrt{T_{sym}} = \sqrt{\frac{SNR}{2}}$

- Please see Handout P comparing 4-PAM and 4-QAM symbol error rates